Goodman’s grue paradox is a spurious paradox. In what follows I will argue that this alleged paradox turns on an ambiguity. For Goodman in formulating his famous paradox draws upon two different evidential claims. I will identify and disentangle the two evidential claims. Next, I will demonstrate that once we keep these two evidential claims separate there is no ground to raise the paradox.

I proceed as follows:

In section (I) INTRODUCTORY REMARKS I explain why Goodman’s grue paradox is a key issue for theory of justification;

In section (II) A SPURIOUS PARADOX I present my disambiguation strategy in resolving Goodman’s paradox;

In section (III) GOODMAN’S SWEEPING TWIST ON CARNAP’S GROUND I come up with historical explanation of what gave rise to the ambiguity pervading Goodman’s paradox.

1. Introductory Remarks

Among the most pertinent objections against formal theories of justification I would list the following:

1) in the case of formal theories of justification substantive conditions of justification reduce to truth conditions;

2) the form of inference is not sufficient for justification of the conclusion.

Short comments: objection 1) is basically motivated by the fact that the traditional tripartite definition of justification as Justified True Belief holds beliefs as justification–bearers. On most formal theories of justification, however, beliefs fall out. And once we dispense with beliefs as justification–bearers it is hard to come up with a satisfactory concept of evidence, i.e. one
that is not reducible to truth conditions. I only mention it here for the sake of completeness, but will not discuss it in detail.

Objection 2) turns on counter-arguments which present unjustified conclusions of inferences having a purportedly justificatory form. Among these counter-arguments the most outstanding one is Goodman’s paradox. In fact, most mainstream epistemologists, like R. Chisholm, A. Goldman, K. Lehrer and E. Sosa derive from Goodman’s paradox the conclusion that in general no formal theory of justification, and in particular no theory of confirmation, can be a feasible candidate for a theory of justification.

This conclusion as well as Goodman’s new riddle of induction turn on a confusion which I will now discuss in some detail.

2. A Spurious Paradox

Shortly, Goodman’s paradox employs a queer predicate ‘grue’ which is defined as follows:

is either observed before $t$ and is green or is observed after $t$ and is blue.

Suppose that $t$ is still ahead of us. Since all thus far observed emeralds have been green it follows that the prediction that

$G$ the next emerald to be observed is green

is equally well supported as the prediction that

$Q$ the next emerald to be observed is grue $\dagger$.

However, we would expect the next emerald to be green, not grue. Goodman argues that $G$ and $Q$ are equally well supported by the evidence, because at present both predicates are co-extensive; $\ddagger$ hence, all thus far observed emeralds can be described as green, but can also be described as grue.

Purely syntactic or semantic categories do not constitute a sufficient ground to draw a distinction between the two predicates. We have to appeal to pragmatic terms—so argues Goodman—namely, to the past record of the usage of both predicates and this is where we could locate the asymmetry between them: it is predicate ‘green’ that was predominantly used in the predictions that have been put forward thus far. So, our expectation that the next emerald will be green rather than grue is merely an apparent effect of this prevailing asymmetry, or, ‘entrenchment’ as Goodman called it.

Where is the ambiguity then? Suppose I construe a fairly self-sufficient robot to inspect all gems found on the Earth. It uses two different sets of criteria—‘A’ and ‘B’—to group the stones of the category ‘Z’ it samples. After long enough period of time the robot issues a report which says that the sample gems ‘Z’ it has gathered so far equally well fit ‘A’ and ‘B’ descriptions. Therefore the prediction

$\dagger$ As E. Sober correctly points out there are in fact two different formulations of Goodman's paradox: singular-predictive (‘the next emerald to be observed is…’) and a general-hypothetical (‘all emeralds are…’). The argument I elaborate here for the singular-predictive formulation fits also – mutatis mutandis – the general-hypothetical formulation.

$\ddagger$ By ‘extensive’ I mean here that the semantic value of the formula containing this predicate combinatorially depends on the semantic values of its constituents.
that the next gem ‘Z’ will be ‘A’ and that the next gem will be ‘B’ are equally well supported by the evidence gathered by the robot. Is there anything paradoxical about this prediction? Surely not. A better illustration would be to compare two sex-checkers who select male and female chickens. Suppose they both use two possibly different sets of individuating properties of selection of male chickens and they both are equally successful. However, the criteria of selection are not articulate. Therefore, we do not understand the criteria they apply during the selection. Is it paradoxical to claim that there is more than one way to tell male chickens from the rest?

These examples are not an argument yet, but merely a loose analogy. Assume—following Goodman in this respect—that

\[ B \] all we know about the predicates ‘green’ and ‘grue’ within the domain of discourse set out to convey all and only thus far observed emeralds is that the two are co-extensive.\(^3\)

True, G and Q are equally well supported by the evidence. However, if all we know about ‘green’ and ‘grue’ is that they are co-extensive given the colours of the thus far observed emeralds, then there is no ground to rise an expectation that it is G rather than Q that we would normally predict to be more probable.

In fact—by the axiom of extensionality—‘green’ and ‘grue’ constitute the same property, and this is all we could say about them, if we are not to exceed the relevant knowledge at our disposal which remains informative insofar as merely extensional attributes of the predicates are concerned. On the background knowledge B\(^i\), however, the two predicates are equally well confirmed. Suppose:

\[
\begin{align*}
\text{cG } &\text{ c(EGa, B\(^i\))}; & \text{ where EGa – the next emerald is green} \\
\text{cQ } &\text{ c(EQa, B\(^i\))}; & \text{ where EQa – the next emerald is grue}
\end{align*}
\]

Since EGa \& B\(^i\) \implies EQa \& B\(^i\), then by a simple application of the axioms of probability\(^5\) we have that:

\[
\text{cG Q }\text{ c(EGa, B\(^i\))} = \text{c(EQa, B\(^i\))}.
\]

Moreover, on any evidence compatible with the background knowledge B\(^i\) it is also true that

\[
\begin{align*}
\text{cG Q } , e &\text{ c(EGa, B\(^i\) \& eG)} = \text{c(EQa, B\(^i\) \& eQ)}; \\
\text{where eG/Q – evidence formulated in terms of G/Q}
\end{align*}
\]

For if eG and eQ are compatible with B \&, then eG \implies eQ.

If we assume that the relevant background knowledge consists merely of B\(^i\), then there is no paradox in the fact that G and Q are equally well supported. What is counterintuitive, however, is that our actual background knowledge exceeds B \&. For we know that—despite the fact that at present for any domain of objects G and Q could be construed as co-extensive—these are not identical properties. Why? Roughly, because they mean different things. G means ‘is green’, while Q means ‘is observed before t\(_0\) and is green or later and is blue’. Therefore, even if there is a

\(^3\) It would be perhaps better not to use ‘green’ but rather a predicate co-extensive with it, e.g., ‘grees’.

\(^4\) In order to avoid confusion with the specific meanings of the functions ‘c’ and ‘p’ ascribed to them by T.A.F. Kuipers (Kuipers 2000), I use these symbols in more traditional way to distinguish between the confirmation function in general (‘c’), and the standard probability function (‘p’).

\(^5\) From the axioms it directly follows that if a \equiv b, then p(a, c) = p(b, c).
domain of discourse, or ‘a possible world’, in which both predicates are co-extensive, other domains of discourse, or possible worlds, are readily available in which both predicates are not co-extensive. For instance, if we choose $t_0$ as 8th May 1946, then all predictions $E_{Qa}$ put forward after $t_0$ have turned out to be false, and all emeralds discovered after $t_0$ can no longer be described as ‘grue’.

Thus, our actual background knowledge entails that

$$ D \land G \text{ and } Q \text{ are in fact different properties.} $$

Are we then back to Goodman’s paradox? Not at all. Because our actual background knowledge contains another important item, i.e. we know that $S$ the colour of gems supervenes on their chemical composition and microphysical structure.

The relevant items of our actual background knowledge might be summarised thus:

$$ B^+ = D \land S $$

It is trivial then that

$$ B^+ \rightarrow \neg E_{Qa}. $$

Therefore,

$$ \neg E_{Qa} \land B^+ \equiv B^+. $$

And since we can safely assume that the chemical composition and physical microstructure of emeralds does not change over time$^6$, it follows that $E_{Ga}$ is consistent with $S$, while $E_{Qa}$ entails $\neg S$, i.e. the proposition that colour does not supervene on the chemical composition and physical microstructure of gems$^7$:

$$ E_{Qa} \rightarrow \neg S. $$

From what has been said in the preceding paragraphs it follows then that

$$ B^+ \rightarrow \neg E_{Qa}. $$

$$ \neg E_{Qa} \land B^+ \equiv B^+. $$

And thus we obtain that

$$ c(\neg E_{Qa}, B^+ ) = 1 $$

and, accordingly

$$ c(\neg E_{Qa}, B^+ ) = 0. $$

Since $cQ^+$ cannot be conditionalised on any evidence, then

$$ cQ^+ = cE_{Qa}, B^+ \equiv 0. $$

On the other hand $E_{Ga}$ is compatible with $B^+$, so

$$ \neg(B^+ \rightarrow \neg E_{Qa}), $$

and

$$ cE_{Ga}, B^+ > 0. $$

$^6$ This assumption is taken for granted in the formulation of the new riddle of induction, for otherwise it will not have the flavour of ‘being paradoxical’, i.e. it will be nothing astonishing to attribute the same probability to $G$ and to $Q$.

$^7$ The Goodman-type predicates, and specifically ‘$E_{Q}$’, do not lend support to counterfactuals like ‘Had this gem been an emerald, it would be green’, or ‘Had this gem a colour other than green, it would not have been emerald’. And this makes them incoherent with law-like statements as $S$; cf. also (Sober 1994, p. 234).
On the assumption that the actual background knowledge contains the items specified as $B^+$, then the paradox is also dissolved. We know that $Q$ must be false as we recognize it to be an absurd prediction, inconsistent with the available knowledge.

However, if we mix up the two cases, namely $cG^-Q^-$ and $cQ^+, cG^+$, then we have the apparent paradox that on the one hand the predictions $Q$ and $G$ are confirmed to the same extent by the available evidence, while on the other, $G$ is more probable than $Q$.

To conclude, the solution proposed here may be perceived as a dilemma which disambiguates Goodman’s purported paradox:

$$\text{DD} \quad \text{either you fail to take into account the evidence on how—}$$

in the case of emeralds—‘green’ and ‘grue’ relate to time, and therefore you also fail to notice the ‘paradoxical’ character of the new riddle of induction, or, you bring the relevant evidence to the fore, and you simply infer that there is no paradox.

### 3. Goodman’s Sweeping Twist on Carnap’s Ground

Goodman’s paradox was first formulated (Goodman 1946) as a counter-example to Carnap’s projected system of inductive logic. In a subsequent series of papers Goodman and Carnap confronted their standpoints and the discussion was brought to an end soon after it had begun. Moreover, they never revived it; even worse—both failed to mention it in their subsequent and very influential texts (Goodman 1954/1983; Carnap 1950). Why?

There are no explicit answers. However, what they actually did is a clear indication of the reasons we seek. For Carnap there was no point in further discussion with Goodman. In his own opinion the strongest argument a philosopher might provide has a form of a logical construction. The best argument against Goodman’s attack on the possibility of the systems of quantitative inductive logic would simply be to construct such systems.

Goodman’s standpoint also became more radical as in (Goodman 1954/1983) he generalised the paradox. It no longer was a mere counterexample to inductive logic, but it had the status of the new riddle of induction, i.e. a new and more profound version of the old and persistent philosophical problem of induction.

Thus since the publication of (Goodman 1954/1983) onwards Goodman presents his paradox as a general epistemological dilemma rather than a problem immanent to a yet non-existent quantitative inductive logic. But this is not the end of the story. Even in the new guise the paradox is predominantly acknowledged as

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8 Goodman 1946; Carnap 1947; Goodman 1947; Carnap 1948.

9 The historical explanation to follow is by no means intended as a self-standing argument. Moreover, the disambiguation strategy presented in the preceding section does not turn on the details of this explanation.

10 Carnap 1950.
an argument demonstrating the impossibility of a purely formal theory of justification. It is apparent then that despite Goodman’s efforts to the contrary inductive logic remains the focus of the paradox. And this fact provides also an explanation why in the formulation of the new riddle of induction Goodman played with two different evidential claims.

Carnap called his own system ‘quasi-semantic’, because the primitive predicates were intended to be uninterpreted, unless the context of application would not be specified. On the one hand, the primitive predicates were devoid of meaning and purely ‘extensional’, while on the other they should have been treated as meaningful and ‘intensional’, because the system in which they functioned was not complete until applied to a specific domain of discourse.

This apparent ambiguity in Carnap’s formulation of the system of inductive logic was inherited by Goodman’s paradox. For on the one hand the predicates like ‘green’ and ‘blue’ were supposed to be a part of the purely formal system of inductive logic, while on the other they are well known predicates of natural language. Therefore, in claiming that Q and G are equally well confirmed Goodman ignores the meanings of the colour predicates which occur in G and Q, and ignores the fact that these predicates are no longer uninterpreted and that they fit a specific context of application. Moreover, he ignores the fact that colour predicates cannot stand alone, and that a part of the same context of application is occupied by predicates which describe the chemical structure of gems. Thus, Goodman violates one of the fundamental methodological principles that Carnap sets out for the application of the systems of inductive logic: once the set of primitive predicates is established to fit a given domain of discourse, it has to be complete, i.e. it has to contain all predicates which describe the relevant properties of the objects of this domain of discourse. And in the case of such properties as colour it is apparent that predicates describing chemical and physical properties of the objects are relevant.

Suppose—as Goodman later on did himself—that the new riddle of induction has a general epistemological import and that it constitutes a challenge for any inductive inference. It still remains true that the riddle undermines a project of constructing a quantitative system of inductive logic. For the sake of the present argument we can put aside the problem whether Goodman’s paradox indeed constitutes ‘a new riddle of induction’, and simply identify Goodman’s ‘paradox’ with a counterargument against quantitative inductive logic. The objection apparently takes for granted Carnap’s differentiation between the system of inductive logic, on the one hand, and methodological principles which accompany it, on the other. Therefore, it can be shortly summarized as Goodman’s dilemma:

GD  either you fail to take into account the evidence on how—in the case of emeralds—to differentiate between the predicates ‘green’ and ‘grue’, and therefore G and Q on logical

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11 This kind of argument was first advanced by R. Chisholm in (Chisholm 1966). Almost all mainstream epistemologists accept a version of this argument; see (Goldman 1986; Lehrer 1974; Sosa 1991).

12 For details of Carnap’s systems of inductive logic see (Carnap 1950).
grounds alone cannot but have the same degree of confirmation, or, you bring the relevant evidence to the fore, violating thus your own methodological principles.

The violation of methodological principles results from the fact that if any piece of evidence were to become a part of the system of inductive logic, then such a system would contain empirical claims, and therefore obviously fail to be any sort of logic\textsuperscript{13}.

Surely, Goodman would argue that there is more to his paradox than GD alone. For the claim in GD that

\begin{quote}

either you fail to take into account the evidence on how—in the case of emeralds—to differentiate between the predicates ‘green’ and ‘grue’, and therefore G and Q on logical grounds alone cannot but have the same degree of confirmation

\end{quote}

is supposed not only to be true, but also paradoxical. In view of the argument exposed in section 1 of this paper I find the ‘paradoxical’ character of this claim implausible.

\textsuperscript{13} This characteristically holds for Sober’s “background beliefs” which surpass “the testimony of past observations”; cf. (Sober 1994, p. 236).
References


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