Influence of microwave fields on electron transport through a quantum dot in the presence of direct tunneling between leads

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We consider the time-dependent electron transport through a quantum dot coupled to two leads in the presence of the additional overdot (bridge) tunneling channel. By using the evolution operator method together with the wide-band limit approximation we derived the analytical formulas for the quantum dot charge and current flowing in the system. The influence of the external microwave field on the current and the derivatives of the current with respect to the gate and source-drain voltages has been investigated for a wide range of parameters. The most characteristic feature of including the additional overdot tunneling channel is an asymmetric behavior of the function describing the dependence of the average current vs the gate voltage or the differential conductance vs the source-drain voltage.

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I. INTRODUCTION

Electronic transport in mesoscopic systems has been at the focus of experimental and theoretical interest during the last decade due to recent development in fabrication of small electronic devices and their interesting equilibrium and nonequilibrium properties. Especially interesting are the transport properties of a quantum dot (QD) under the influence of external time-dependent fields. The high-frequency signals may be applied to a QD and the time-dependent fields will modify the tunneling current. New effects have been observed and theoretically described, e.g., photon-assisted tunneling through small quantum dots with well-resolved discrete energy states,1–3 photon-electron pumps,4–6 and others. One can investigate the current flowing through a QD under periodic modulation of the QD electronic structure7 or periodic (nonperiodic) modulation of the tunneling barriers8 and electron energy levels in both (left and right) electron reservoirs8 (see also Refs. 9 and 10).

The progress of nanomaterials science has enabled the experimental study of the phase coherence of the charge carriers in many mesoscopic systems. The asymmetric Fano line shapes11 are observed whenever resonant and nonresonant scattering paths interfere. In some nanostructures, e.g., in single-electron transistors, the Fano resonances in the conductance were observed,12 which imply that there are two paths for transfer of electrons between a source and a drain. Especially the recent experimental and theoretical study with a low-temperature scanning tunneling microscope (STM) of the single magnetic atom deposited on a metallic surface showed the asymmetric Fano resonances in the tunneling spectra.12–15 The STM measurements indicate that in tunneling of electrons between STM tip and a surface with a single impurity atom two different paths are present. The electrons can tunnel between the tip and the adsorbate state and directly between the tip and the metal surface. The electronic transport through a QD coupled to the electron reservoirs within a model with two electron tunneling channels was considered in Ref. 16 and it was shown that transport of electrons through both channels leads to an asymmetric shape of the zero-bias voltage conductance curves, which is typical behavior for a Fano resonance resulting from constructive and destructive interference processes for electrons transmitted through both channels.

In all papers mentioned above and relating to the electron transport through a QD with the additional (bridge) transmission channel the external fields were not applied and the considered systems were driven out of equilibrium only by means of a dc voltage bias. In this paper we address the issue of a QD with a bridge channel between a source and a drain driven out of equilibrium by means of a dc voltage bias and additional time-dependent external fields. In this manner, our paper can be seen as generalization of Ref. 9 to the case of a QD with the additional bridge channel in the presence of external microwave fields which are applied to the dot and two leads, respectively. In literature, different theoretical approaches have been developed to treat the time-dependent, nonequilibrium electron transport processes in the mesoscopic systems. It seems that the most popular is the nonequilibrium Green’s function method. However, these Green’s functions depend on the time arguments and for nontrivial quantum models it is a rather difficult task to calculate them. In our treatment of the time-dependent tunneling through a mesoscopic system we use the evolution operator technique (e.g., Refs. 17 and 18). The final expressions for the QD charge and the current flowing in the system can be described in terms of the corresponding matrix elements of the evolution operator. In our earlier work19 we have considered the similar problem solving numerically the corresponding sets of the differential equations satisfied by the matrix elements of the evolution operator. Due to the complexity and a large number of these equations we considered only a very limited number of interesting cases although we were able to take into consideration the electronic structure of the lead energy bands and the specific time dependence of the QD-lead barriers. Here we give the analytical expressions for the QD charge and current assuming so-called the wide-band limit approximation and the time-independent strength of the QD-lead barriers. As a result, due to these analytical forms,
we are able to analyze the required characteristics of the considered system for the very broad class of parameters. Additionally, due to the final forms given for some matrix elements of the evolution operator it is possible to build up the expressions for the QD charge or current in the form of the perturbation series.

In the following section we present the model and formalism and give the resulting expressions for equations for the corresponding matrix elements of the evolution operator. In Sec. III we obtain the approximate solutions for all required matrix elements and give the final forms for the QD charge and the current flowing in the system. The results and a brief discussion are presented in Sec. IV and conclusions are given in Sec. V.

II. MODEL AND CALCULATION METHOD

We model the QD coupled to the left and right electron reservoirs with the additional bridge tunneling channel between them by the usually used Hamiltonian $H = H_1 + V$, where

$$H_1 = \sum_{k_a} \epsilon_{k_a} (t) a_{k_a}^{\dagger} a_{k_a} + \epsilon_{d} (t) a_{d}^{\dagger} a_{d},$$

$$V = \sum_{k_a} V_{k_a} (t) a_{k_a}^{\dagger} a_{d} + \text{H.c.} + \sum_{k_L, k_R} V_{k_L, k_R} (t) a_{k_L}^{\dagger} a_{k_R} + \text{H.c.}$$

The operators $a_{k_a}^{\dagger}$ ($a_{k_a}$) and $a_{d}^{\dagger}$ ($a_{d}$) are the annihilation (creation) operators of the electron in the lead ($\alpha = L, R$) and in the QD, respectively. The couplings between QD and lead states and between both lead states are denoted by $V_{k_a}$ and $V_{k_L, k_R}$, respectively. For simplicity, the dot is characterized only by a single level $\epsilon_{d}$ and we have neglected the intradot electron-electron Coulomb interaction. We assume the case in which there exist microwave fields applied to the leads and QD. Although in most theoretical treatments of photon-assisted electron tunneling it is assumed that the driving field equals the applied external field, the situation is more complicated and the internal potential can be different from the applied potential. One of the consequences should be, e.g., the asymmetry between the corresponding left and right sidebands. However, the main features of the time-dependent transport remain unchanged so in our treatment we assume that system considered here is described by $\epsilon_{k} (t) = \epsilon_{k} + \Delta_{\alpha} \cos \omega t$, $\epsilon_{d} (t) = \epsilon_{d} + \Delta_{d} \cos \omega t$, i.e., the energy levels of the leads and QD are driven by the ac field with the frequency $\omega$ and the amplitudes $\Delta_{\alpha}$ and $\Delta_{d}$, respectively.

We describe the dynamical evolution of the charge localized on the QD and the current flowing in the system in terms of the time evolution operator $U(t, t_0)$ (in the interaction representation, $\hbar = 1$) which satisfies the equation (e.g., Refs. 17 and 18)

$$i \frac{\partial}{\partial t} U(t, t_0) = \tilde{V}(t) U(t, t_0),$$

where

$$\tilde{V}(t) = U_0(t, t_0) V(t) U_0^\dagger(t, t_0),$$

$$U_0(t, t_0) = T \exp \left( i \int_{t_0}^{t} dt' H_1(t') \right).$$

Here we assume that the interaction between QD and leads and between both leads is switched on in the distant past $t_0$, i.e., $V_{k_a}(t)$ and $V_{k_L, k_R}(t)$ are equal to zero for $t \leq t_0$ and take constant values for $t > t_0$.

The QD charge and currents flowing in the system can be obtained from the knowledge of the appropriate matrix elements of the evolution operator $U(t, t_0)$. The QD charge is given as follows (cf. Ref. 18):

$$n_d(t) = n_d(t_0) + \sum_{k_a} n_{k_a}(t_0) \left| U_{d k_a}(t, t_0) \right|^2,$$

where

$$U_{d k_a}(t, t_0) = \langle d | U(t, t_0) | k_a \rangle$$

denote the matrix elements of $U(t, t_0)$ calculated within the basis functions containing the electron single-particle functions of the leads and QD, $| k_L \rangle$, $| k_R \rangle$, and $| d \rangle$, respectively. $n_d(t_0)$ and $n_{k_a}(t_0)$ represent the initial filling of the corresponding single-particle states.

The tunneling current flowing, e.g., from the left lead into the QD and the right lead, $j_{L}(t)$, can be obtained from the time derivative of the total number of electrons in the left lead, $j_{L}(t) = -e \sum_{k} \frac{\partial n_{k_L}(t)}{\partial t}$ (cf. Ref. 9), where

$$n_{k_L}(t) = \sum_{k_L} n_{k_L}(t) = \sum_{k_L} \left[ n_{d}(t_0) \left| U_{k_L d}(t, t_0) \right|^2 + \sum_{q_L} n_{q_L}(t_0) \left| U_{k_L q_L}(t, t_0) \right|^2 + \sum_{k_R} n_{k_R}(t_0) \left| U_{k_L k_R}(t, t_0) \right|^2 \right].$$

Let us begin with the calculations of the QD charge $n_d(t)$. Then we have to calculate the matrix elements $U_{d k_a}(t, t_0)$ and $U_{d k_a}(t, t_0)$. Using the identity operator $I = | d \rangle \langle d | + \sum_{k_a} | k_a \rangle \langle k_a |$ the following set of coupled equations can be obtained from Eq. (3):

$$\frac{\partial}{\partial t} U_{dd}(t, t_0) = -i \sum_{k_a} \tilde{V}_{k_a}(t) U_{d k_a}(t, t_0),$$

$$i \frac{\partial}{\partial t} U_{d k_a}(t, t_0) = \tilde{V}_{k_a}(t) U_{d k_a}(t, t_0),$$

$$i \frac{\partial}{\partial t} U_{k_L d}(t, t_0) = \tilde{V}_{k_L d}(t) U_{k_L d}(t, t_0),$$

$$i \frac{\partial}{\partial t} U_{k_L q_L}(t, t_0) = \tilde{V}_{k_L q_L}(t) U_{k_L q_L}(t, t_0),$$

$$i \frac{\partial}{\partial t} U_{k_L k_R}(t, t_0) = \tilde{V}_{k_L k_R}(t) U_{k_L k_R}(t, t_0).$$
The formal solution of Eq. (9) written in the form

$$\frac{\partial}{\partial t} U_{\tilde{k},dd}(t,t_0) = -i \int_{t_0}^{t} dt' \tilde{V}_{\tilde{k},dd}(t') U_{\tilde{k},dd}(t',t_0)$$

where

$$\tilde{V}_{\tilde{k},dd}(t)\approx \tilde{V}_{\tilde{k},dd}(t) = V_{\tilde{k},dd} \exp \left[ i (\Delta_{d} - \Delta_{a})(\sin \omega t - \sin \omega t_0)/\omega \right] .$$

It is easy to show that the equation for $U_{dd}(t,t_0)$ can be written as follows:

$$\frac{\partial}{\partial t} U_{dd}(t,t_0) = -i \int_{t_0}^{t} dt' \left( K(t,t') U_{dd}(t',t_0) \right)$$

where

$$K(t,t') = \sum_{\tilde{k}_a} V_{\tilde{k},dd}(t) \tilde{V}_{\tilde{k},dd}(t')$$

and the similar equation can be written for $L_{\tilde{k},dd}(t,t')$.

The formal solution of Eq. (9) written in the form

$$U_{\tilde{k},dd}(t,t_0) = -i \int_{t_0}^{t} dt' \tilde{V}_{\tilde{k},dd}(t') U_{\tilde{k},dd}(t',t_0)$$

and performing similar calculations to those described above, one obtains for $U_{dd}(t,t_0)$ [and similar equation for $U_{dd}(t,t_0)$ by interchanging $L\leftrightarrow R$]

$$U_{Ld}(t,t_0) = -i \int_{t_0}^{t} dt_1 \tilde{V}_{L}(t_1) U_{dd}(t_1,t_0)$$

$$+ \sum_{j=2}^{\infty} (-i)^j \sum_{L_1,R_1,L_2,R_2,...} \int_{t_0}^{t_1} dt_1 \cdots \int_{t_0}^{t_{j-1}} dt_j$$

$$\times \tilde{V}_{L,R_1}(t_1) \tilde{V}_{L_1,R_2}(t_2) \cdots \tilde{V}_{L_{j-1}}(t_j) U_{dd}(t_j,t_0).$$

Here we have introduced the abbreviated form for the vector $\tilde{k}_a$ and replaced it by $\alpha$ and $\tilde{V}_{\tilde{k},dd}(t)\approx \tilde{V}_{\tilde{k},dd}(t)$. The equation for $U_{Rd}(t,t_0)$ can be obtained from Eq. (17) by interchanging $L\leftrightarrow R$. Inserting these expressions for $U_{Ld}(t,t_0)$ and $U_{Rd}(t,t_0)$ into Eq. (13) one obtains the closed integrodifferential equation for $U_{dd}(t,t_0)$:

$$\frac{\partial}{\partial t} U_{dd}(t,t_0)$$

$$= -i \int_{t_0}^{t} dt_1 \tilde{K}(t,t_1) U_{dd}(t_1,t_0)$$

$$+ \sum_{j=2}^{\infty} (-i)^j \sum_{L_1,R_1,L_2,R_2,...} \int_{t_0}^{t_1} dt_1 \cdots \int_{t_0}^{t_{j-1}} dt_j$$

$$\times \tilde{V}_{L_1}(t_1) \tilde{V}_{L_2}(t_2) \cdots \tilde{V}_{L_{j-1}}(t_j) U_{dd}(t_j,t_0)$$

$$(\text{the second term with the change } L\leftrightarrow R).$$

Equation (18) together with the expressions for $\tilde{V}_{L}(t)$ and $\tilde{V}_{LR}(t)$, Eqs. (11) and (12), and for $\tilde{K}(t,t_1)$ written as

$$\tilde{K}(t,t_1) = \sum_{\tilde{k}_a} |V_{\tilde{k},dd}|^2 \tilde{D}_a(t-t_1) \exp [i e_d(t-t_1)]$$

$$+ i(\Delta_d - \Delta_a)(\sin \omega t - \sin \omega t_1)/\omega,$$

where $D_a(t)$ a Fourier transform of the $a$th lead density of states, gives exact, closed equation for $U_{dd}(t,t_0)$. Here we have assumed that $\tilde{V}_{\tilde{k}_a}$ does not depend on the wave vector $\tilde{k}_a$ and then the similar assumption will be made for $\tilde{V}_{\tilde{k}_a}$.

Under the wide-band limit (WBL) approximation (e.g., Ref. 9) this equation can be analytically solved and such solutions will be considered later. Formally, solving Eq. (18) and inserting its solution into Eq. (17), the solutions for $U_{Ld}(t,t_0)$ and $U_{Rd}(t,t_0)$ can be obtained. The function $U_{Ld}(t,t_0)$ is needed in the calculations of the first term of $f_L(t)$ [see Eq. (7)].

In order to calculate $n_d(t)$ we still need $U_{dd}(t,t_0)$. Writing down the set of closed equations for $U_{dd}(t,t_0)$, $U_{Rd}(t,t_0)$, and $U_{LR}(t,t_0)$ [obtained on the basis of Eq. (3)] and performing similar calculations to those described above, one obtains for $U_{dd}(t,t_0)$ [and similar equation for $U_{dd}(t,t_0)$ by interchanging $L\leftrightarrow R$]
The analytical solutions of these equations under the WBL approximation will be discussed in the following section.

For calculation of \( n_{\xi}(t) \) one needs the functions \( U_{LR}(t,t_0) \), \( U_{RL}(t,t_0) \), \( U_{L_1L_2}(t,t_0) \), and \( U_{RL}(t,t_0) \). The first two functions are given in Eqs. (17) and (21), respectively, and \( U_{L_1L_2}(t,t_0) \) should be calculated from the set of coupled equations for \( U_{dL_1}(t,t_0) \), \( U_{L_1L_2}(t,t_0) \), and \( U_{RL}(t,t_0) \) obtained from Eq. (3). The result is as follows:

\[
U_{L_1L}(t,t_0) = \delta_{L_1L} + \sum_{j=2,4,...}^{\infty} (-i)^j \sum_{R_1L_2R_3,...R_{j-1}} \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdot \cdot \cdot \int_{t_0}^{t_{j-1}} dt_j \bar{V}_{L_1R_1}(t_1) \bar{V}_{L_2R_2}(t_2) \cdot \cdot \cdot \bar{V}_{R_{j-1}L}(t_j) \\
- i \int_{t_0}^{t} dt_1 \bar{V}_{L_1}(t_1) U_{dL}(t_1,t_0) + \sum_{j=2}^{\infty} (-i)^j \sum_{R_1L_2R_3,...R_{j-1}} \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdot \cdot \cdot \int_{t_0}^{t_{j-1}} dt_j \bar{V}_{L_1R_1}(t_1) \bar{V}_{L_2R_2}(t_2) \cdot \cdot \cdot \bar{V}_{R_{j-1}L}(t_j) \bar{V}_{L_1L_2}(t,t_0) \bar{V}_{L_1L_2}(t,t_0) \\
\times \bar{V}_{L_1R_1}(t_1) \bar{V}_{L_2R_2}(t_2) \cdot \cdot \cdot \bar{V}_{R_{j-1}L}(t_j) U_{dL}(t_j,t_0),
\]

where \( U_{dL}(t,t_0) \) is given by solving Eq. (20) with the replacement \( L \rightarrow R \).

### III. Analytical Solutions in the WBL Approximation

In order to calculate the QD charge \( n_j(t) \) or current \( j_j(t) \) one has to solve, in the first step, the integrodifferential equations satisfied by \( U_{dL}(t,t_0) \), Eq. (18), and \( U_{dRL}(t,t_0) \), Eq. (20). The other needed functions \( U_{dL}(t,t_0) \), \( U_{LR}(t,t_0) \), and \( U_{L_1L_2}(t,t_0) \) can be obtained from Eqs. (17), (21), and (22) inserting into them \( U_{dL}(t,t_0) \) and \( U_{dRL}(t,t_0) \) and performing multiple time and \( \vec{k} \)-vector integrations. Unfortunately, it is a very difficult task to solve the integrodifferential equations and to perform these integrals in a general case when the leads are characterized by some density of state curves. Here, we use the WBL approximation, under which all multiple time and \( \vec{k} \)-vectors integrals can be performed without difficulties. The WBL approximation has been widely used in calculation of many properties of mesoscopic systems (e.g., Refs. 2,3,5,9 and 10). It is justified under the conditions that the QD energy-level linewidth is much smaller than the bandwidth of the leads and that the density of states and hopping matrix elements vary slowly with energy. Furthermore, as we are not going to consider the case of the QD energy level lying close to the edges of the lead energy band or lying close to some singular structure present in the lead energy band, then application of the WBL approximation should be fully justified (see, e.g., Ref. 20). The conditions under which we perform our calculations are satisfied in most experimental constructions of mesoscopic systems. As a check, we have performed the direct but time consuming numerical integration of Eqs. (8)–(10) (and similar equations for other functions) for the rectangular lead density of states and did not find any differences in the results for the time-averaged QD charge or the currents flowing in the consid-
Let us consider the equation for the matrix element $U_{dd}(t,t_0)$, Eq. (18). The function $K(t,t')$, Eq. (19), is approximated in WBL as follows

$$K(t,t_1) = \sum_\alpha |V_{a\alpha}|^2 \int_{-\infty}^{\infty} d\epsilon \mathcal{D}_\alpha(\epsilon) \exp[-i\epsilon(t-t_1)]$$

$$\times \exp[i\epsilon_d(t-t_1) + i(\Delta_d - \Delta_a)]$$

$$\times (\sin \omega t - \sin \omega t_1) / \omega \Rightarrow \sum_\alpha |V_{a\alpha}|^2 \mathcal{D}_\alpha(\epsilon) \frac{2\pi}{\omega} \delta(t-t_1),$$

where the lead density of states $\mathcal{D}_\alpha(\epsilon)$ was replaced by the rectangular density of states with the effective bandwidth $\mathcal{D}_\alpha$.

Using similar approximation in calculations of the multiple integrals present in Eq. (18) one obtains

$$\frac{\partial}{\partial t} U_{dd}(t,t_0) = \left( -\Gamma_a - \frac{2V^2}{D} \sum_{j=1}^{\infty} (-i\pi V_{RL} / D)^j \right) U_{dd}(t,t_0),$$

where $\Gamma_a = 2\pi V^2 / D$, $\Gamma_L = \Gamma_R = \Gamma$, $D_L = D_R = D$, and $V = V_a$.

Assuming $\pi V_{RL} / D < 1$ the series can be summed up and finally the equation for $U_{dd}(t,t_0)$ reads

$$\frac{\partial}{\partial t} U_{dd}(t,t_0) = -C_1 U_{dd}(t,t_0),$$

where $C_1 = (2\pi V^2 / D) \gamma (1 + i \pi V_{RL} / D)$.

Similarly, Eq. (20) becomes

$$\frac{\partial}{\partial t} U_{da}(t,t_0) = -i\tilde{V}_{da}(t)/(1 + i\pi V_{RL} / D) - C_1 U_{da}(t,t_0).$$

The solutions of Eqs. (25) and (26) are as follows:

$$U_{dd}(t,t_0) = \exp[-C_1(t-t_0)],$$

$$U_{da}(t,t_0) = -i(1 + i\pi V_{RL} / D) \int_{t_0}^{t} dt_1 \tilde{V}_{da}(t_1)$$

$$\times \exp[-C_1(t-t_1)].$$

The QD charge $n_d(t)$ can be easily obtained, Eq. (6). It can be verified that the first term of Eq. (6) tends to zero as $t - t_0 \rightarrow \infty$ and finally for the QD charge we have

$$n_d(t_0) = \frac{1}{(1 + (V_{RL} \pi / D)^2)} \sum_\alpha \frac{\Gamma_a}{2\pi} \int d\epsilon f_\alpha(\epsilon) |A_\alpha(\epsilon,t)|^2.$$

where

$$A_\alpha(\epsilon,t) = -i \int_{t_0}^{t} dt_1 \exp[-i(\epsilon_d - \epsilon)(t-t_1) - i(\Delta_d - \Delta_a)]$$

$$\times (\sin \omega t - \sin \omega t_1) / \omega - C_1(t-t_1).$$

In the limit of vanishing bridge over the QD, Eq. (29) reproduces the result of Ref. 9.

The current $j_L(t)$ flowing from the left lead into the QD and the right lead is calculated from the evolution of the total number of electrons in the left lead [see Eq. (7)], and one can read (we set $e = 1$)

$$j_L(t) = -2\text{Re} \left( \sum_{k_L} n_d(t_0) U_{k_L,d}(t,t_0) \frac{d}{dt} U_{k_L,L}(t,t_0) \right.$$

$$\left. + \sum_{k_L \neq k_R} n_d(t_0) U_{k_L,d}(t,t_0) \frac{d}{dt} U_{k_R,L}(t,t_0) \right.$$
\begin{equation}
\langle j_L(t) \rangle = \frac{1}{\pi} \frac{2\chi^2}{(1+x^2)} \int (f_L(\epsilon) - f_R(\epsilon)) d\epsilon - \frac{\Gamma}{1+x^2} \langle n_d(t) \rangle \\
- \text{Im} \left\{ \frac{1-x^2}{(1+x^2)(1+i)^2} \frac{\Gamma}{\pi} \int d\epsilon f_L(\epsilon) (A_L(\epsilon, t)) \right\} \\
- \text{Re} \left\{ \frac{2x}{(1+x^2)(1+i)^2} \frac{\Gamma}{\pi} \int d\epsilon f_R(\epsilon) (A_R(\epsilon, t)) \right\}.
\end{equation}

(36)

In the vanishing bridge channel case Eq. (36) coincides with the results of Ref. 9:

\begin{equation}
\langle j_L^{V_RL=0}(t) \rangle = - \Gamma \langle n_d(t) \rangle - \frac{\Gamma}{\pi} \int d\epsilon f_L(\epsilon) \text{Im}(A_L(\epsilon, t)).
\end{equation}

(37)

The current \( \langle j_L(t) \rangle \), Eq. (36), flowing from the left lead to the central region and to the right lead (through the bridge channel) is the superposition of four terms. The first term corresponds to the current between two leads and this term is not disturbed by the QD. The form of the second and third terms is the same as for \( \langle j_L^{V_RL=0}(t) \rangle \), Eq. (37), except for the renormalization constants due to the additional tunneling channel. Note that some additional renormalizations also occur due to \( V_{RL} \) which enters into the expression for \( n_d(t) \), Eq. (29), and for \( A_d(\epsilon, t) \), Eq. (30). The last term of Eq. (36) is the interference term due to the simultaneous tunneling through two channels.

IV. RESULTS AND DISCUSSION

Here we show the numerical results of the time-averaged current \( \langle j_L(t) \rangle \) and its derivatives with respect to the QD energy level position and the chemical potential \( \mu_L \) (or equivalently, with respect to the gate and source-drain voltages) for different sets of parameters which characterize our system. We assume the temperature \( T = 0 \) K and the time average of the density quantities \( f(t) \) is defined by

\begin{equation}
\langle f(t) \rangle = \frac{1}{\tau \tau_0} \int \frac{d\tau'}{\tau_0} f(t'),
\end{equation}

and because \( f(t) \) is a periodic function of time, we average it over the period \( 2\pi/\omega \). We take the chemical potential of the right lead \( \mu_R \) as the energy measurement reference point, \( \mu_R = 0 \). As the potential drop between the left and right leads is given by \( \mu_L - \mu_R = eV_{\perp} \) and \( V_{\perp} \) is the measured voltage between a source and a drain, the derivatives of the current \( \langle j_L(t) \rangle \) with respect to \( \mu_L \) will correspond to the derivatives \( d\langle j_L(t) \rangle/dV_{\perp} \) usually measured in experiments. In experiments the gate voltage controls the position of the QD energy level \( \epsilon_d \) (regardless of how complicated the relation between the gate voltage and \( \epsilon_d \) is) and for that reason to mimic measurements of the QD charge or current vs the gate voltage we have calculated them vs the position of the QD energy level \( \epsilon_d \).

The values of the hybridization matrix elements \( V_{\perp} \) present in the Hamiltonian do not enter the final expressions for the current or QD charge obtained within the WBL approximation. Usually the effective linewidth \( \Gamma_\alpha = 2\pi \Sigma_{\varepsilon} |V_{\perp}|^2 \partial(\epsilon - \epsilon_L) \) is introduced. However, in our calculations the other hybridization matrix elements appear, \( V_{\perp} \) responsible for the additional tunneling channel for which we should take some values in order to perform numerical calculations. We have taken the values comparable with \( V_{\perp} \) and estimated \( V_{\perp} \) (assuming its \( k \) independence, \( V_{\perp} = V_{\perp} = V \)) using the relation \( \Gamma_\alpha = 2\pi |V_{\perp}|^2/D\alpha, \) where \( D\alpha \) is the \( \alpha \)-lead’s bandwidth and \( D\alpha = 100 \) \( \Gamma_\alpha \) (\( \Gamma_\alpha = \Gamma_\alpha = \Gamma_\alpha \)).

In Fig. 1 we show the average current \( \langle j_L(t) \rangle \) against \( \epsilon_d \) for given values of \( \mu_L \) (beginning from \( \mu_L = -4 \) up to \( \mu_L = 8 \)). The left and right panels correspond to \( V_{RL} = 0 \) and \( V_{RL} = 10 \), respectively, \( \mu_L = 0 \), \( V_\perp = 4, \Delta_L = 2, \Delta_R = 1, \Delta_R = 0, \omega = 2 \).

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At first, we consider the dependence of \( \langle j_L(t) \rangle \) vs \( \epsilon_d \) for given values of the left lead chemical potential \( \mu_L \). In Fig. 1 we present such curves for different values of \( \mu_L \)—the subsequent curves beginning from the lower one correspond to \( \mu_L = -4 \) up to the upper curve (with the step \( \Delta\mu_L = 1 \) obtained for \( \mu_L = 8 \)). The left (right) panel corresponds to \( V_{RL} = 0 \) (\( V_{RL} = 10 \)). In the case of vanishing overdot tunneling channel (the left panel) the current has a simple structure—a single peak localized in the middle between \( \mu_L \) and \( \mu_R \) for smaller values of \( \mu_L \). The width of this peak increases with increasing \( |\mu_L| \) and for greater values of \( |\mu_L| \) the current is almost independent of \( \epsilon_d \) localized inside the energy region between \( \mu_R \) and \( \mu_L \). For nonvanishing overdot tunneling (Fig. 1, the right panel) the curves \( \langle j_L(t) \rangle \) become asymmetric. With increasing source-drain bias, the current possesses greater values in comparison with the \( V_{RL} = 0 \) case due to the direct tunneling between both leads. Note, however, that due to the interference effects the resulting \( \langle j_L(t) \rangle \) curves are asymmetric. The interference effects are most visible for \( \epsilon_d \) lying approximately in the region \( \mu_L, \mu_L \).

In Fig. 2 we show the average current \( \langle j_L(t) \rangle \) vs the left lead chemical potential \( \mu_L \) for several values of \( \epsilon_d \). For vanishing \( V_{RL} \) (the left panel) the corresponding curves are nearly asymmetric with the asymmetry point \( \mu_L = \epsilon_d \). With
increasing $\mu_L$ at fixed $e_d$, the current achieves a constant value depending on the position of the QD energy level $e_d$ with respect to the $\mu_R=0$. It means that electrons which occupy the lead energy levels not too distant from $e_d$ take part in the tunneling process. For greater $\mu_L$, most of the lead energy levels lying far away from $e_d$ are inactive in the tunneling between leads through the QD energy level. However, at nonvanishing $V_{RL}$ (see the right panel of Fig. 2) these lead energy levels are active and the current $\langle j_L(t) \rangle$ vs $\mu_L$ is of much richer structure. The current is nearly linearly growing with the increasing $\mu_L$ (for larger $\mu_L$) because the tunneling through the QD can be neglected compared with the direct tunneling between both leads. The clearly visible interference effects appear only for $\mu_L$, not too distant from $e_d$.

In Fig. 3 we show the derivatives of the average current vs the QD energy level $e_d$, obtained for some values of $\mu_L$. There are the results of the differentiation of curves shown in Fig. 1. Again, the most visible differences between the results obtained for $V_{RL}=0$ and $V_{RL}\neq 0$ are present for the QD energy level $e_d$ localized approximately between chemical potentials of both leads (compare, for example, the curves calculated for $\mu_L=8$).

Figure 4 presents the comparison of the $d\langle j_L(t) \rangle/d\mu_L$ vs $\mu_L$ curves calculated for vanishing $V_{RL}$ (left panels) and for $V_{RL}=10$ (right panels) for two different values of the amplitudes $\Delta_L$ ($\Delta_d=\Delta_L/2$, $\Delta_R=0$). At the vanishing value of $V_{RL}$, the shape of the curves is symmetrical in relation to the values $\mu_L=e_d$ although for greater $\Delta_L$ some shoulders appear on both sides of the corresponding peaks in the distance $\sim \Delta_L/2$ from the curve centers. For nonvanishing $V_{RL}$, the corresponding curves are approximately asymmetric and for large values of $\mu_L$ they tend to constant, nonzero values corresponding to linear increasing of the current at large $\mu_L$. It is interesting that with the increasing amplitudes $\Delta_L$ and $\Delta_d$ very clear structures appear on both sides of the corresponding curves. Note that all these curves can be obtained, for example, from the one calculated for $e_d=0$ and moved along the $\mu_L$ axis by the corresponding value (equal to $e_d$).

Figure 5 shows the dependence of the average current $\langle j_L(t) \rangle$ on the QD energy level $e_d$ and the direct coupling $V_{RL}$ between both leads. The upper and bottom panels corresponds to $\mu_L=8$, the upper (lower) panel corresponds to $\mu_L=-4$ ($\mu_L=2$, $V=4$).

FIG. 2. The average current $\langle j_L(t) \rangle$ against $\mu_L$ for given values $e_d$ (beginning from $e_d=-4$ up to $e_d=8$). The broken curves correspond to $e_d=0$. The left (right) panel corresponds to $V_{RL}=0$ ($V_{RL}=10$). The other parameters are as in Fig. 1.

FIG. 3. The derivatives of the average current against $e_d$ with respect to the QD energy level $e_d$, $d\langle j_L(t) \rangle/d e_d$, for given values of $\mu_L$ (beginning from $\mu_L=-4$ up to $\mu_L=8$). The broken curves correspond to $\mu_L=-4$. The left (right) panel corresponds to $V_{RL}=0$ ($V_{RL}=10$) and the other parameters are as in Fig. 1.

FIG. 4. The derivatives of the average current against $\mu_L$ with respect to $\mu_L$, $d\langle j_L(t) \rangle/d \mu_L$, for given values of $e_d$ (beginning from $e_d=-4$ up to $e_d=8$). The left (right) panels correspond to $V_{RL}=0$ ($V_{RL}=10$) and upper (lower) panels correspond to $\Delta_L=2$ ($\Delta_L=4$). The other parameters are as in Fig. 1.

FIG. 5. The average current $\langle j_L(t) \rangle$ against $V_{RL}$ and $e_d$. The upper (lower) panel corresponds to $\mu_L=8$, $\mu_L=-4$ ($\mu_L=2$, $V=4$).
and the different curves 
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them and the central peak is an integer multiple of the fre-
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Figure 6 presents the average current \( \langle j_L(t) \rangle \) obtained for the oscillating QD energy level at \( V_{RL}=0, 4, 7, \) and 10—broken, thin, thick, and very thick curves, respectively. The panels (a), (b), (c), and (d) correspond to \( \Delta_l=1, 3, 5, \) and 7, respectively. \( \omega =5, \Gamma =1, V =4, \mu_L=0.2, \) and \( \Delta_L=\Delta_R=0. \)

correspond to different values of the amplitude \( \Delta_L \) \( \Delta_d =\Delta_l/2 \). A very distinct transition from the symmetric to nearly antisymmetric behavior of the current vs \( \epsilon_d \) curves is observed with the increasing value of the overdot additional coupling between the leads for both values of \( \Delta_L \). The larger amplitudes of the left lead and QD level oscillations result only in some broadening of the characteristic features of the average current vs \( V_{RL} \) and \( \epsilon_d \) surface, and do not introduce any additional structures (for the range of parameters \( \omega \sim \Gamma \) and \( \mu_L-\mu_R \) not very small).

Figure 6 presents the average current \( \langle j_L(t) \rangle \) against \( \epsilon_d \) for the oscillating QD energy level at \( V_{RL}=0, 4, 7, \) and 10—broken, thin, thick, and very thick curves, respectively. The panels (a), (b), (c), and (d) correspond to \( \Delta_l=1, 3, 5, \) and 7, respectively. \( \omega =5, \Gamma =1, V =4, \mu_L=0.2, \) and \( \Delta_L=\Delta_R=0. \).

equal to the height of the central peak. If we take the addition overdot tunneling channel into consideration, especially for small \( \Delta_d \), the asymmetric shape of the current curve is observed and this asymmetry increases with the increasing strength of the overdot coupling between both leads. With the increasing amplitude \( \Delta_d \) this asymmetry is reduced largely due to the extra, photon-assisted tunneling peaks whose strength increases with the increasing \( \Delta_d \). For sufficiently large values of \( \Delta_d \) and \( V_{RL} \) the functional dependence of the average current on the QD energy level \( \epsilon_d \) is nearly the same for vanishing and nonvanishing overdot tunneling channels. There is only one difference—for large \( V_{RL} \) the corresponding curve is shifted to the higher values due to the direct channel between both leads.

Figures 7 and 8 are devoted to the analysis of the average current \( \langle j_L(t) \rangle \) dependence on the oscillation period \( 2\pi/\omega \) of the external fields. In Fig. 7 we show the overall dependence of \( \langle j_L(t) \rangle \) on \( 2\pi/\omega \) and the QD energy level \( \epsilon_d \). The upper (lower) part of the figure presents the results for the vanishing (nonvanishing) overdot channel between both leads. The most visible differences between averaged currents calculated for \( V_{RL}=0 \) and \( V_{RL} \neq 0 \) can be observed for \( \mu_R=\epsilon_d=\mu_L \), especially for small values of \( 2\pi/\omega \). More detailed analysis of the \( \langle j_L(t) \rangle \) dependence on the oscillation period \( 2\pi/\omega \) is shown for some chosen values of the parameters \( \epsilon_d \) and \( V_{RL} \) in Fig. 8. The thin (thick) curves correspond to \( \epsilon_d=1 \) \( (\epsilon_d=5) \) and the solid (broken) curves correspond to \( V_{RL}=4 \) \( (V_{RL}=0). \) Additionally, we give the
results for two values of the amplitude $\Delta_L$ ($\Delta_d = \Delta_L / 2$, $\Delta_R = 0$), i.e., for $\Delta_L = 5$ and $\Delta_L = 10$, the left and the right parts of Fig. 8, respectively. We observe the characteristic average current oscillations damped with increasing $2 \pi / \omega$ for $e_d$ lying in the central part between the left and right chemical potentials (see also Ref. 9). These oscillations are present for both $V_{RL} = 0$ and $V_{RL} \neq 0$ and are more visible for greater amplitudes $\Delta_d$ and $\Delta_R$ but the maxima and minima of the oscillating average current are localized at the same value of the oscillation period. Note that the existence of the additional overdot tunneling channel results approximately in shifting the corresponding curves to higher values without any additional serious modifications. For the QD energy level lying away from the middle point between the left and right chemical potentials, the average current is still an oscillating function of $2 \pi / \omega$ although these oscillations are less transparent and their oscillation period is much longer.

It is worth to mention that if the voltages of the left and right leads and the gate voltage are shifted by the same amount, the current does not change. Our formulation satisfies this requirement as can be deduced from construction of the functions $\bar{V}_{ij}(t)$, Eqs. (11) and (12). After using the WBL approximation this property of the final result for the quantum dot charge and current is still preserved. The formalism satisfies also other requirement, namely, for the case when the microwave fields are applied to the all parts of the system and are characterized by the same amplitudes, no physical effect exists by external fields. This is clearly seen as the function $A_{ij}(\epsilon, t)$ which enters into Eqs. (29) and (36) depends only on the difference of the corresponding amplitudes.

V. CONCLUSIONS

We have provided a detailed investigation of a QD connected to two leads with an additional overdot tunneling channel. The harmonic external microwave fields were considered as applied to the QD and two leads which result in time dependence of the corresponding QD and lead energy levels. The QD charge and the average current flowing in this system were calculated within the evolution operator technique. The corresponding matrix elements of the evolution operator required for the calculation of the QD charge and current were presented in the form of the infinite series of multiple time integrals of the functions containing the information about the coupling between the QD and leads or in the form of the integrodifferential equation. Applying the WBL approximation we were able to obtain all required evolution operator matrix elements in closed forms and give the final analytical expressions for the QD charge and current. We have performed the extended numerical calculations for the average current and the derivatives of the current with respect to the gate and source-drain voltages. The most spectacular influence of the additional bridge tunneling channel is visible in the $\langle j_L(t) \rangle$ dependence vs the position of the QD energy level at the constant source-drain voltage. Going from the vanishing values of the overdot tunneling channel strength to the nonvanishing one the corresponding curves transform, due to the interference effects, from the symmetric to nearly antisymmetric (Fig. 1). Similar influence of the nonvanishing $V_{RL}$ is visible in the dependence of $\langle d(j_L(t))/d\mu_L \rangle$ vs $\mu_L$ (Fig. 4). The characteristic behavior of the average current vs the QD energy-level position at the small source-drain voltage is observed for the case when the external oscillating field is applied only to the QD (Fig. 6). For the vanishing overdot tunneling channel at the small amplitude $\Delta_d$, the main resonant peak is only observed and with the increasing amplitude $\Delta_d$ the next peaks localized at $e_d$ equal to the multiplicity of $\omega$ appear corresponding to the photon-assisted tunneling. For the nonvanishing overdot tunneling channel and small amplitudes $\Delta_d$, the character of the dependence of the average current on the QD energy level position transforms with increasing $V_{RL}$ from the symmetrical to nearly antisymmetrical behavior. This tendency to the antisymmetrical behavior with increasing $V_{RL}$ at small amplitude $\Delta_d$ is reduced with increasing $\Delta_d$. For sufficiently large amplitude $\Delta_d$ the overall behavior of the average current vs $e_d$ is very similar for different values of $V_{RL}$ and does not manifest the tendency for the asymmetry with increasing $V_{RL}$.

Note added in proof. The similar time-dependent Hamiltonian has recently been studied by Ma et al.\textsuperscript{23} using the nonequilibrium Green’s function method. These authors, however, have mainly been interested in the shot noise induced in the quantum dot.

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